Data analysis: responses at weekly time scales

Jeff White, Wed. 15:20

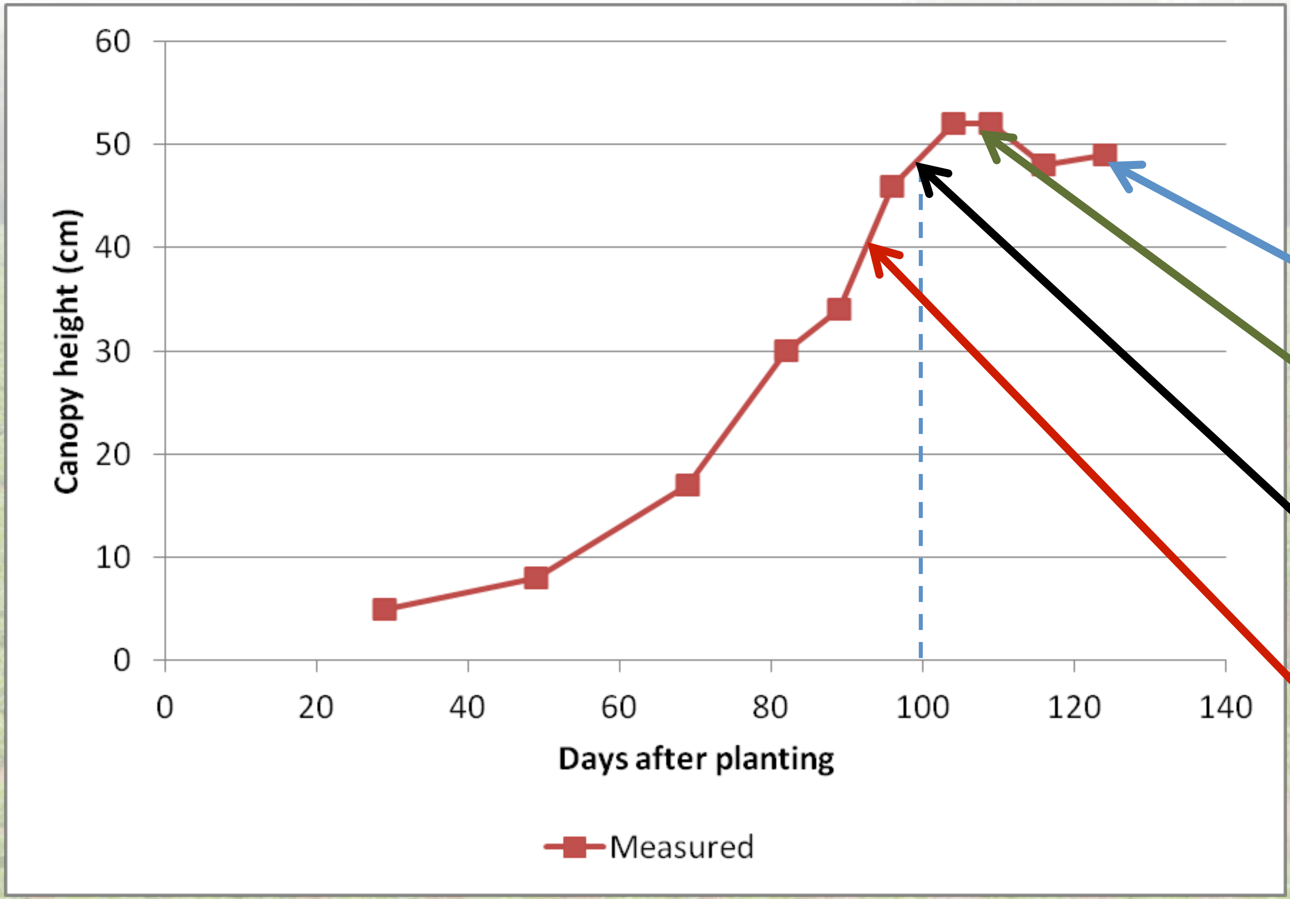
Introduction

- From first 2.5 days of workshop
  - Can acquire data over days using field phenomics systems
  - Can georegister to obtain plot-level data
- How does one make the most efficient use of data?
  - Maximize the information content
  - Minimize error
- Approach for analyzing time series is based on classical growth analysis, 1950s to 1980s

Consider canopy height of spring wheat over time

Field 105, 2013.

Single wheat plot



What might we analyze as the phenotype?

Final height

Maximum height

Height at day 100

Maximum elongation rate

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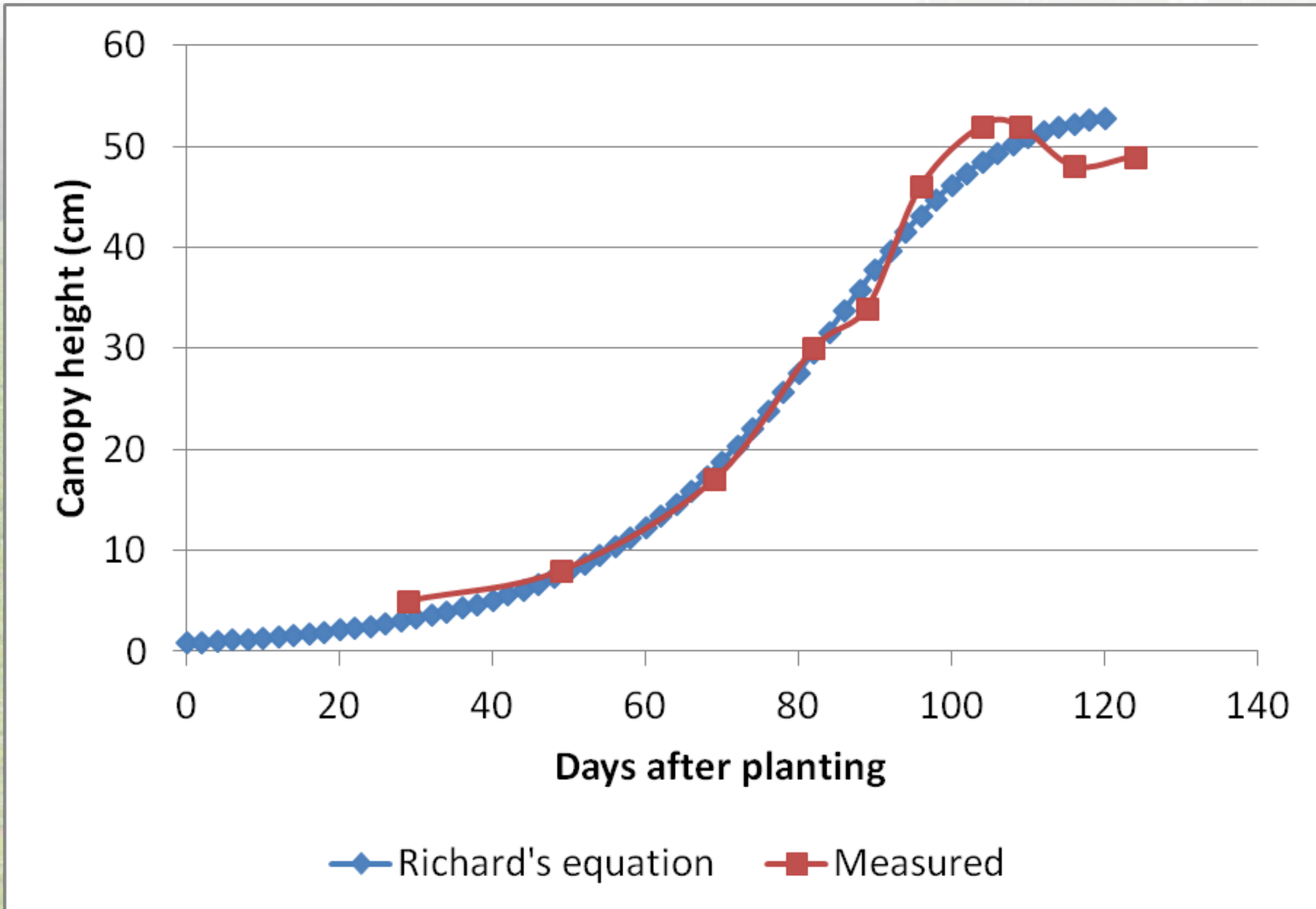
What is the best estimate of growth: the raw or the fitted data?



Fitted curves:

- Reduce sampling error
- Curve must be well chosen

Once fitted, you can estimate any parameter:

Maximum height

Height at day 100

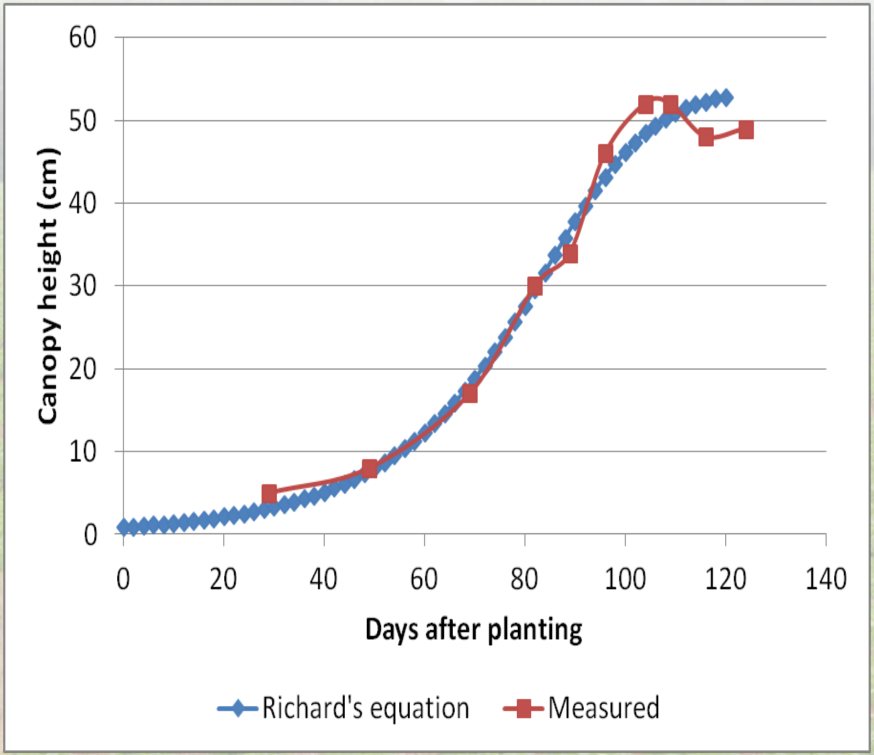
Final height

Maximum elongation rate

Rationale for fitting growth curves

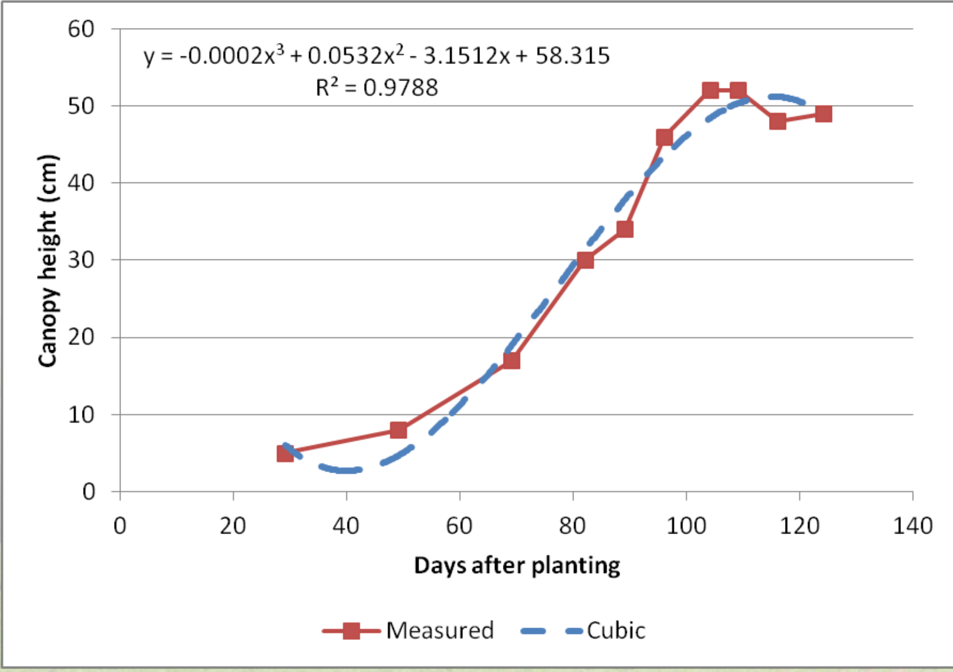
"The rationale behind the use of the fitted functions is then simple: if attempts to assess the reality of growth result in a time series of observations scattered randomly about that reality, then a suitable mathematical function fitted to those observations may be expected to regain much of the clarity with which reality is perceived by the experimenter."

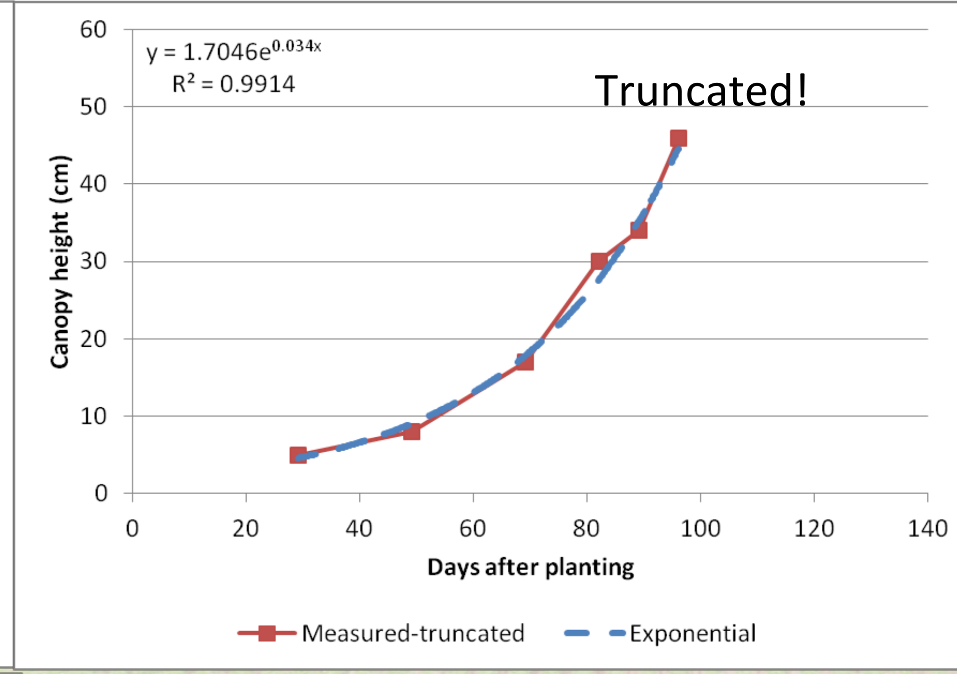
R. Hunt. 1979. Ann Bot 43:245

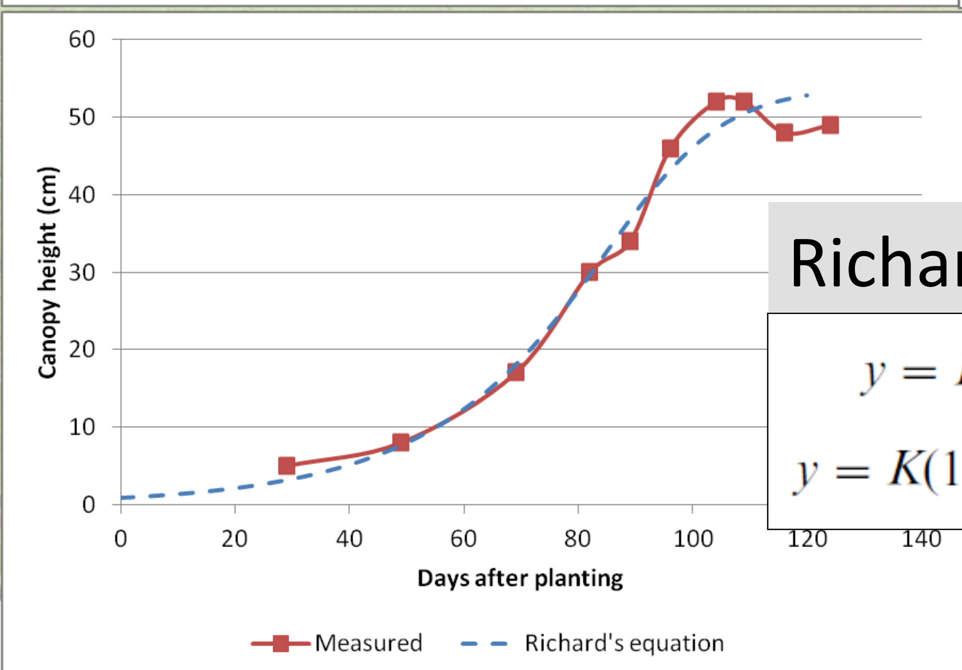


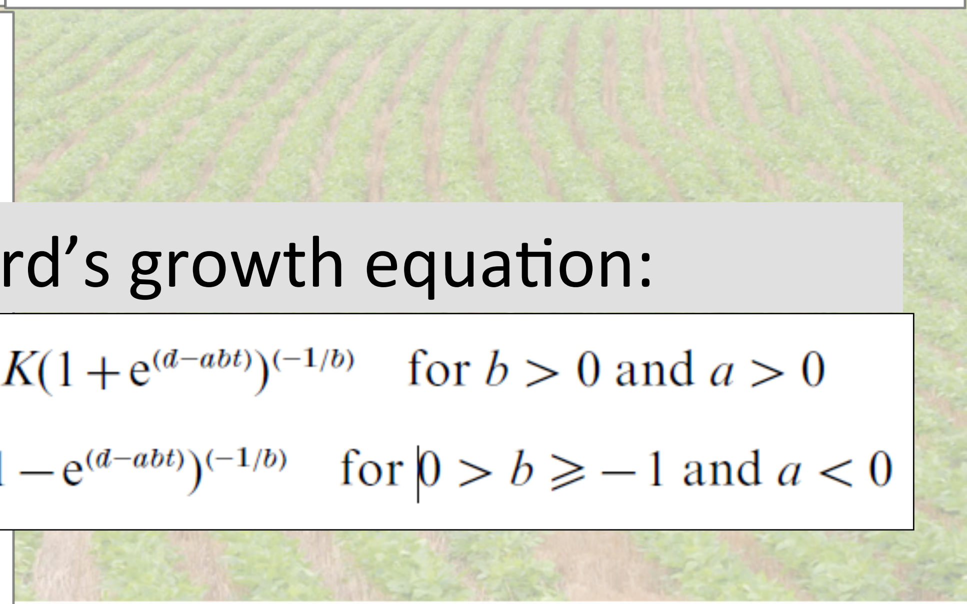
Examples of curves used to fit curves

- Quadratic
- Cubic
- Various exponential equations
- Richard’s equation
- “Cottage industry” of papers describing “improved curves” (see references)









Richard’s growth equation:

$$y = K(1 + e^{(d-abt)})^{(-1/b)} \quad \text{for } b > 0 \text{ and } a > 0$$

$$y = K(1 - e^{(d-abt)})^{(-1/b)} \quad \text{for } 0 > b \geqslant -1 \text{ and } a < 0$$

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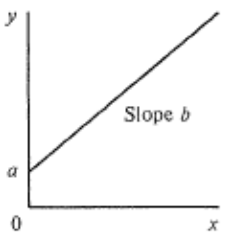
Source: Milthorpe & Moorby. 1979
Crop Physiology. 2nd ed.

Some commonly used functions

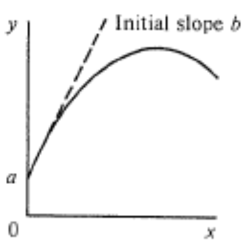
Below are given some of the functions which are widely used in growth studies and to relate processes to different factors. More information on these and other functions, and on the problems of fitting them, can be obtained in Pollard, J. H. (1971). *A handbook of numerical and statistical techniques*. Cambridge University Press; and Landsberg, J. J. (1977). Some useful equations for biological studies. *Expl Agric.* **13,** 273–80.



1. Polynomial

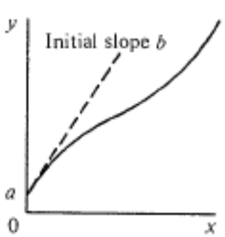
(*a*) *Linear*

Function	$y = a + bx$
Slope	$dy/dx = b$
Version for fitting	$y = \bar{y} + b\ (x - \bar{x})$



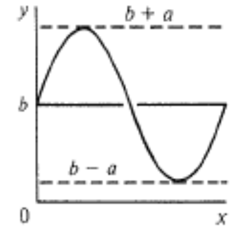
(*b*) *Quadratic*

Function	$y = a + bx + cx^2$
Slope	$dy/dx = b + 2cx$
Version for fitting	$y = \bar{y} + b(x - \bar{x}) + c(x^2 - \bar{x}^2)$ (or orthogonal functions)



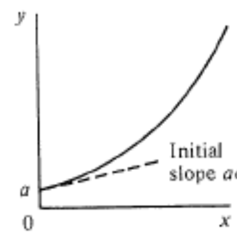
(*c*) *Cubic*

Function	$y = a + bx + cx^2 + dx^3$
Slope	$dy/dx = b + 2cx + 3dx^2$
Version for fitting	$y = \bar{y} + b(x - \bar{x}) + c(x^2 - \bar{x}^2) + d(x^3 - \bar{x}^3)$ (or orthogonal functions)



2. Sine

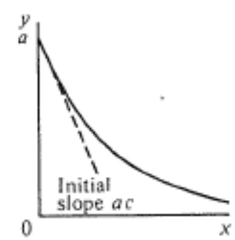
Function	$y = a \sin x + b$
Slope	$dy/dx = a \cos x$
Version for fitting	$y = a \sin x + b$, where x is transformed to radians



3. Exponential

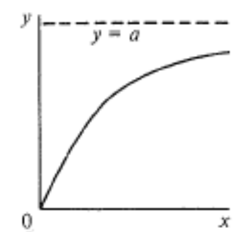
(*a*) *Growth*

Function	$y = ae^{cx}$ $(c > 0)$
Slope	$dy/dx = ace^{cx} = cy$
Version for fitting	$\ln y = \ln a + cx$ $= \overline{\ln y} + c(x - \bar{x})$



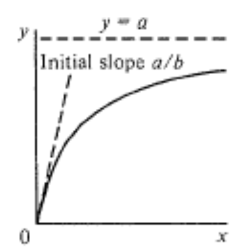
(*b*) *Decay*

Function	$y = ae^{-cx}$ $(c > 0)$
Slope	$dy/dx = -ace^{-cx} = -cy$
Version for fitting	$\ln y = \ln a + cx$



(*c*) *Reciprocal*

Function	$y = ae^{-c/x}$ $(c > 0)$
Slope	$dy/dx = -ace^{-c/x}/x^2$
Version for fitting	$\ln y = \ln a - c/x$



4. Hyperbolic

Function	$y = ax/(b + x)$
Slope	$dy/dx = ab/(b + x)^2$
Version for fitting	$1/y = 1/a + b/(ax)$

Time and environmental effects

- We observe crops growing and developing over *time*
- But crops are responding to:
  - Temperature
  - Water status
  - Nitrogen status
  - Etc.
- Can we improve analysis by analyzing based on environment instead of time?
  - Growth = f(time)
  - Growth = f(weather, management, soils, etc.)
    - "Physiological time"
    - Full ecophysiological modeling (see lecture on inverse modeling)

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"Physiological time"

- Cumulative sum
  - Temperature: Heat units, growing degree days, thermal units
  - Photothermal time: Temperature + photoperiod
  - Water stress days
- What is really going on?
  - Summing over time ~ integrating a rate over time
  - Simplest model

$$dG/dt = R \times f(T)$$

where: G = growth trait

R = potential rate (assume constant for a genotype)

f(T) = temperature effect on rate

Simple model: a closer look

$dG/dt = R \times f(T)$

where: G = growth trait

R = potential rate

$f(T)$ = temperature effect on rate

$$\int dG = \int R \times f(T)\, dt$$

$$G = R \int f(T)\, dt$$

$$G \approx R \sum f(T)\, \Delta t$$ ← summation over time (e.g., days)

Analyses based on sums of "physiological time" are actually applying simple rate-based (process) models

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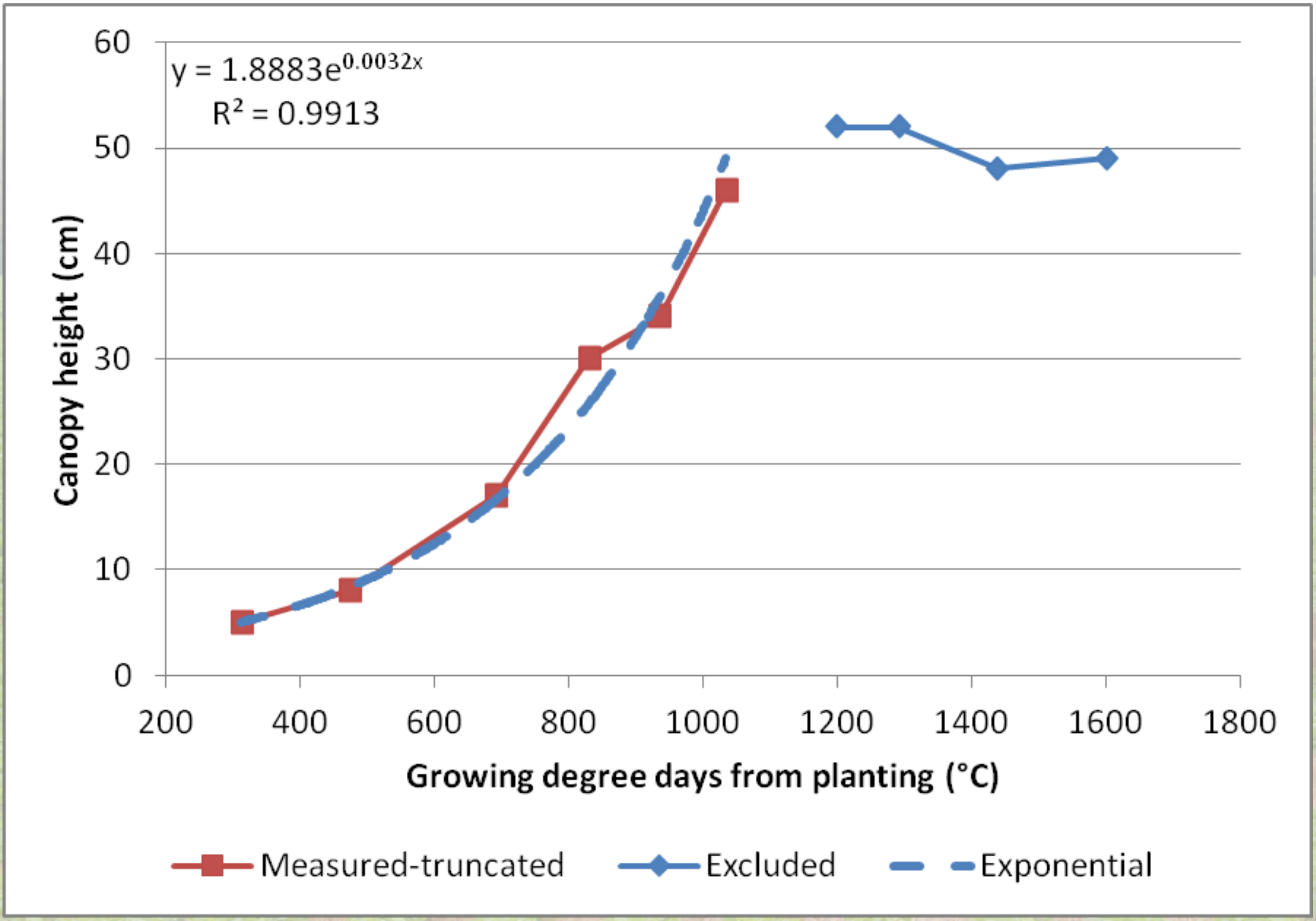
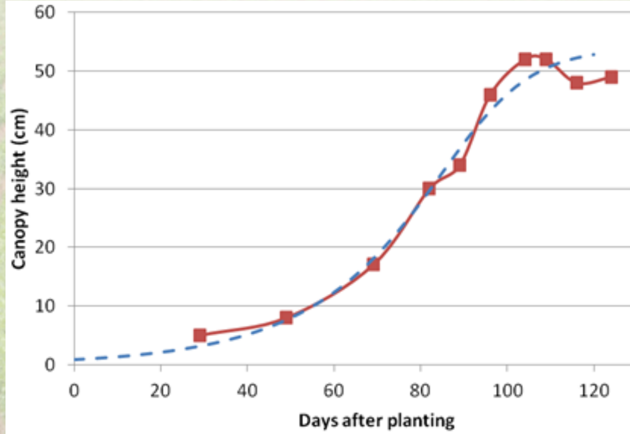
Example for wheat height



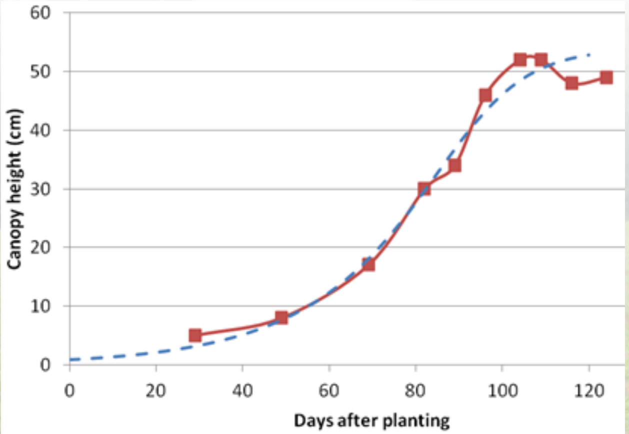
- Assume base temperature of 0°C
- Sum from planting onward
- No improvement relative to DAP (R^2 = 0.9914)
- For:
  - Single location
  - Single planting date

Estimating parameters from curves

- Phenotypic values at specific times
  - Plant height at anthesis
  - Plant height at X DAP even if samples taken on other dates (useful for very large trials)
- Fundamental phenotypic traits
  - Maximum height
  - Maximum growth rate
- Timing of important events
  - Time maximum value is first reached
  - Time of maximum growth or extension



Estimating parameters from curves



Relative growth rates (RGR)

H = height

t = time

$$RGR = 1/H \times dH/dt$$

or

$$RGR = (H_b - H_a)/(0.5 * (H_b + H_a)) * /(t_b - ta)$$

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The approach is readily extended to other variables

- Classical growth analysis

  W = crop dry weight

  LAI = leaf area index

  Crop growth rate, CGR = dW/dt

  Net assimilation rate, NAR = CGR/LAI

  Leaf Area Duration LAD = ∫LAI dt

- Use NDVI or other variables to estimate W and LAI

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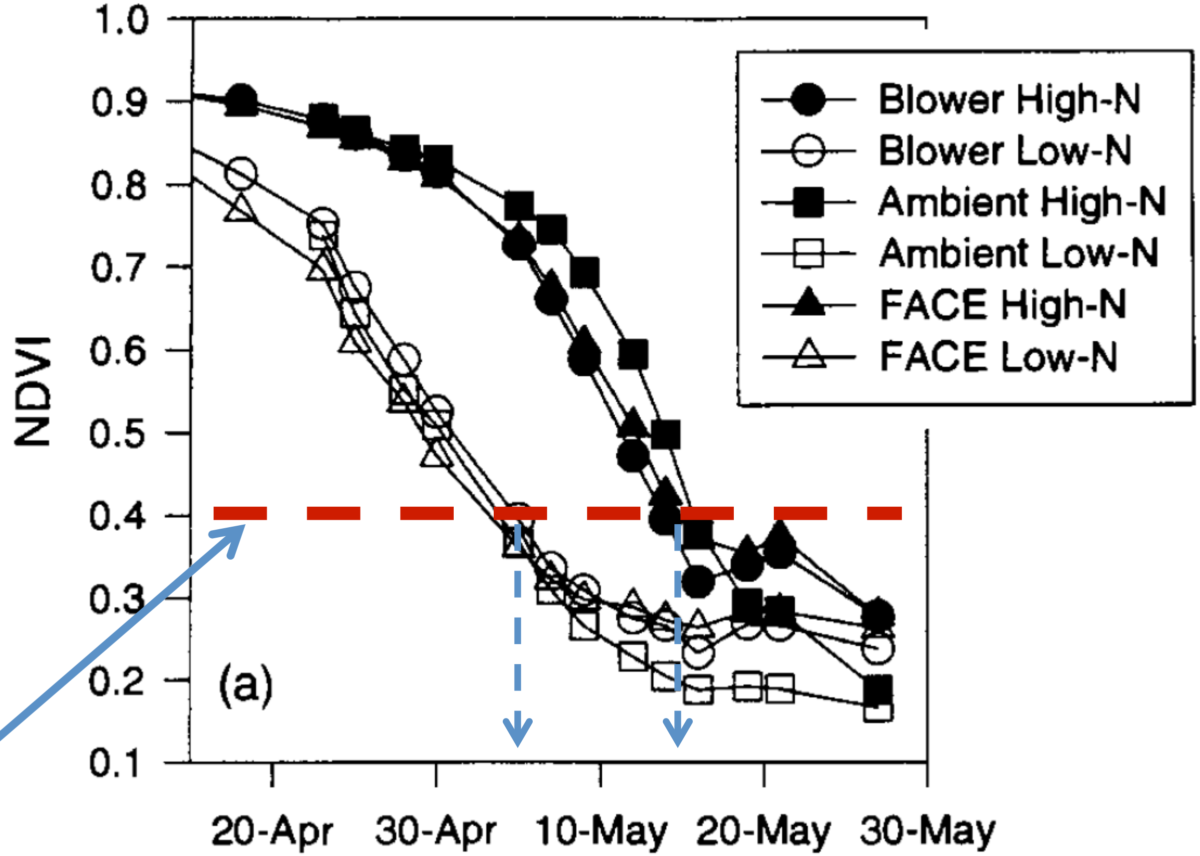
Other shapes of curves

Late-season NDVI in spring wheat in Maricopa

- Maximum rate of decline
- Date that NDVI < Limit value, *X*



Source: Adamsen 1999 Crop Sci 39: 719-724.

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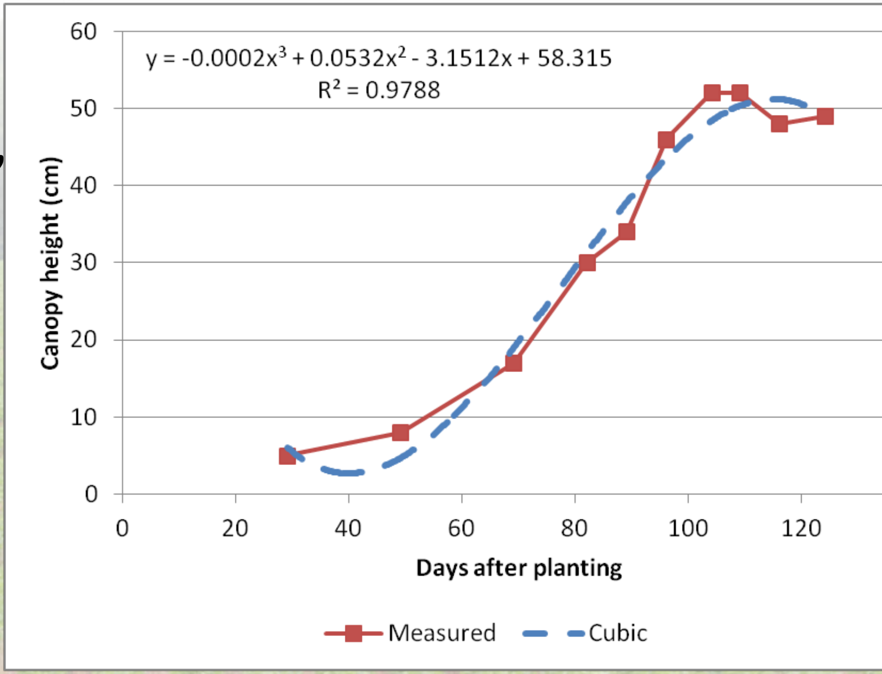
Curve fitting per se

- A poorly selected curve can bias estimated values, especially at extremes
- Fitting is more difficult than it looks
  - R^2 often very high
  - Need to apply "sensibility checks"



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Conclusion

- Fitting growth curves
  - Reduces effect of sampling error
  - Allows inclusion of simple environmental effects
    - Temperature as “physiological time”
    - Other stresses
  - Allows calculation of numerous parameters besides the value at *X* days after planting
  - Appropriate curve may be difficult to identify
- *Opinion*: Complicated curve-fitting approaches are less promising than explicitly developing process-based model and applying model techniques → ***Next lecture and exercise!***

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References

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- Landsberg J. 1977. Some useful equations for biological studies. Experimental Agriculture 13:273-286.
- Wu R., Ma C.-X., Yang M.C., Chang M., Littell R.C., Santra U., Wu S.S., Yin T., Huang M., Wang M. 2003. Quantitative trait loci for growth trajectories in Populus. Genetical research 81:51-64.
- Yin X., Goudriaan J., Lantinga E.A., Vos J., Spiertz H.J. 2003. A flexible sigmoid function of determinate growth. Annals of Botany 91:361-371.

Lecture presented at:
Field Phenomics Workshop
Maricopa Agricultural Center
Maricopa, AZ
March 16-19, 2015

For further material relating to field phenomics and information on future workshops, visit fieldphenomics.org.

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